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On paranorm intuitionistic fuzzy *I*-convergent sequence spaces defined by compact operator

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ABSTRACT

The purpose of this paper is to introduce paranorm intuitionistic fuzzy I-convergent sequence spaces defined by compact operator and study the fuzzy topology on the said spaces. We defined more general type of paranorm intuitionistic fuzzy I-convergent sequence $S^I_{(\mu,\nu)} = (T)(p)$ and $S^I_{(0,\nu)} = (T)(p)$ spaces by using compact operators. Moreover, we established some topological properties concerning with those spaces.

1. Introduction

After the pioneering work of Zadeh (1965), a huge number of research papers have appeared on fuzzy theory and its applications as well as fuzzy analogues of the classical theories. Fuzzy set theory is a powerful hand set for modelling uncertainty and vagueness in various problems arising in field of science and engineering. It has a wide range of applications in various fields: population dynamics (Barros et al., 2000), chaos control (Fradkov and Evans, 2005), computer programming (Giles, 1980), nonlinear dynamical system (Hong and Sun, 2006), etc. Fuzzy topology is one of the most important and useful tools and it proves to be very useful for dealing with such situations where the use of classical theories breaks down. The concept of intuitionistic fuzzy normed space (Saadati and Park, 2006) and of intuitionistic fuzzy 2-normed space (Mursaleen and Lohani, 2009) is the latest developments in fuzzy topology. Khan et al. (2014, 2015, 2017) and Khan and Yasmeen (2016a,b,c) studied the intuitionistic fuzzy zweier I-convergent sequence spaces defined by paranorm, modulus function and Orlicz function.

The notion of statistical convergence is a very useful functional tool for studying the convergence problems of numerical problems/matrices (double sequences) through the concept of density. The notion of I-convergence, which is a generalization of statistical convergence (Fast, 1951; Esi and Özdemir, 2016; Mursaleen and Mohiuddine, 2009a;b;2010; Hazarika and Mohiuddine, 2013; Alotaibi et al., 2014; Mohiuddine and Lohani, 2009; Mursaleen et al., 2010) was introduced by Kostyrko et al. (2000) by using the idea of I of subsets of the set of natural numbers ℕ and further studied in (Nabiev et al., 2007). Recently, the notion of statistical convergence of double sequences x = (x_{ij}) has been defined and studied by Edely (2003), and for fuzzy numbers by Savas (2004), Mursaleen et al. (2016). Quite recently, Das et al. (2008) studied the notion of I and *I*-convergence of double sequences in ℝ.

We recall some notations and basic definitions used in this paper.

Definition 1.1: A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be continuous t-norm, if the following hold:
1. $*$ is associative and commutative,
2. $*$ is continuous,
3. $x * 1 = x$ for all $x \in [0,1]$,
4. $x * y \le z * w$ whenever $x \le z$ and $y \le w$ where $x, y, z, w \in [0,1]$.

Example 1.1: Define $x * y = x.y$ where the usual multiplication is. Then it can be shown that $*$ is a continuous t-norm.

Definition 1.2: A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is said to be continuous t-norm, if it satisfies the following properties:

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1. $\diamond$ is associative and commutative,
2. $\diamond$ is continuous,
3. $x \diamond 0 = x$ for all $x \in [0,1]$,
4. $x \diamond y \leq z \diamond w$ whenever $x \leq z$ and $y \leq w$ where $x, y, z, w \in [0,1]$.

Example 1.2: $x \diamond y = \min\{x + y, 1\}$ is a continuous t-norm.

Definition 1.3: Let X be a non-empty set. A subsets I of X is said to be an ideal if,

1. $A, B \in I \Rightarrow A \cup B \in I$;
2. $A \in I, B \subseteq A \Rightarrow B \in I$.

The above properties are called additivity and hereditary respectively. An Ideal I is called non-trivial if $X \neq I$.

Definition.1.4: Let X be a non-empty set. Then $\mathcal{F} \subseteq 2x$ is said to be filter on X if,

1. $\emptyset \notin \mathcal{F}$,
2. $A, B \in \mathcal{F} \Rightarrow A \cap B \in \mathcal{F}$,
3. $A \in \mathcal{F}$ and $A \subseteq B \Rightarrow B \in \mathcal{F}$.

To each ideal I, a filter $\mathcal{F}$ is associated defined as $\mathcal{F}(I) = \{M \subseteq X : Mc \in I\}$.

Definition 1.5: Let $I \subseteq 2\mathbb{N}$ be a non-trivial ideal in $\mathbb{N}$. Then a sequence $x = (x_k)$ is said to be I-convergent to a number $L \in \mathbb{R}$ if, for every $\epsilon > 0$, the set

$\{k \in \mathbb{N}: |x_k - L|\} \in I$.

Definition 1.6: Let $I \subseteq 2\mathbb{N}$ be a non-trivial ideal in $\mathbb{N}$. Then a sequence $x = (x_k)$ is said to be I-Cauchy if, for each $\epsilon > 0$, there exists a number $N = N(\epsilon)$ such that the set $\{k \in \mathbb{N}: |x_K - x_N| \geq \epsilon\} \in I$.

Definition 1.7: Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function:

$\mu_A : A \rightarrow [0,1]$

and $\mu_A(x)$ is called as the degree of membership of element x in fuzzy set A for each x in X.

Definition 1.8: The five-tuple $(X, \mu, \nu, *, \diamond)$ is said to be an intuitionistic fuzzy normed space(IFNS) if X is a vector space, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and μ, ν are fuzzy sets on $X \times (0, \infty)$ satisfying the following conditions for every $x, y \in X$ and $s, t > 0$ (Khan et al., 2015):

a) $\mu(x,t) + \nu(x,t) \leq 1$,
b) $\mu(x,t) > 0$,
c) $\mu(x,t) = 1$ if and only if $x = 0$
d) $\mu(\alpha x, t) = \mu\left(x, \frac{t}{|\alpha|}\right)$ for each $\alpha \neq 0$
e) $\mu(x,t) * \mu(y,s) \leq \mu(x + y, t + s)$,
f) $\mu(x,t) * (0,\infty) \rightarrow [0,1]$ is continuous,
g) $\lim_{t\to\infty} \mu(x,t) = 1$ and $\lim_{t\to 0} \mu(x,t) = 0$,
h) $\nu(x,t) \leq 1$,
i) $\nu(x,t) = 0$ if and only if $x = 0$,
j) $\nu(\alpha x, t) = \nu\left(x, \frac{t}{|\alpha|}\right)$ for each $\alpha \neq 0$,
k) $\nu(x,t) \diamond \nu(y,s) \geq \nu(x + y, t + s)$,
l) $\nu(x, .): (0,\infty) \rightarrow [0,1]$ is continuous
m) $\lim_{t\to\infty} \nu(x,t) = 0$ and $\lim_{t\to 0} \nu(x,t) = 1$.

In this case (μ, ν) is called an intuitionistic fuzzy norm.

Definition 1.9: Let $(X, \mu, \nu, *, \diamond)$ be IFNS then the sequence $x = (x_k)$ is said to be convergent to continuous $L \in X$ with respect to the intuitionistic fuzzy norm (μ, ν) if, for every $\epsilon > 0$ and $t > 0$, there exists $k_0 \in \mathbb{N}$ such that $\mu(x_k - L, t) > 1 - \epsilon$ and $\nu(x_k - L, t) < \epsilon$ for all $k \geq k_0$. In this case we write $(\mu, \nu) - \lim x = L$.

Definition 1.10: Let $(X, \mu, \nu, *, \diamond)$ be IFNS then the sequence $x = (x_k)$ is said to be Cauchy sequence with respect to the intuitionistic fuzzy norm (μ, ν) if, for every $\epsilon > 0$ and $t > 0$, there exists $k_0 \in \mathbb{N}$ such that $\mu(x_k - x_l, t) > 1 - \epsilon$ and $\nu(x_k - x_l, t) < \epsilon$ for all $k, l \geq k_0$.

Definition 1.11: Let K be the subset of natural numbers $\mathbb{N}$. Then the asymptotic density of K, denoted by $\delta(K)$, is defined as $\delta(K) = \lim_n \frac{1}{n}|\{k \leq n: k \in K\}|$, where the vertical bars denote the cardinality of the enclosed set.

Definition 1.12: A number sequence $x = (x_k)$ is said to be statistically convergent to a number ℓ if, for each $\epsilon > 0$, the set $K(\epsilon) = \{k \leq n: |x_k - \ell|\} \geq \epsilon$ has asymptotic density zero, i.e. $\lim_n \frac{1}{n}|\{k \leq n: |x_k - \ell|\} \geq \epsilon\}| = 0$, In this case we write $st - \lim x = \ell$.

Definition 1.13: A number sequence $x = (x_k)$ is said to be statistically Cauchy convergent if, for every $\epsilon > 0$, there exists a number $N = N(\epsilon)$ such that

$$\lim_n \frac{1}{n}|\{j \leq n: |x_j - x_N|\} \geq \epsilon\}| = 0.$$

The concepts of statistical convergence and statistical Cauchy for double sequences in intuitionistic fuzzy normed spaces have been studied by Mursaleen et al. (2010).

Definition 1.14: Let $I \subseteq 2\mathbb{N}$ be a non-trivial ideal and $(X, \mu, \nu, *, \diamond)$ be an IFNS then the sequence $x = (x_k)$ of elements of X is said to be I-convergent to $L \in X$ with respect to the intuitionistic fuzzy norm (μ, ν) if for every $\epsilon > 0$ and $t > 0$, the set
$\{k \in \mathbb{N} : \mu(x_k - L, t) \geq 1 - \epsilon \text{ or } \nu(x_k - L, t) \leq \epsilon\} \in I$.

In this case L is called the I-limit of the sequence (x_k) with respect to the intuitionistic fuzzy norm (μ, ν) and $I_{(\mu,\nu)} - \lim(x_k) = L$.

Definition 1.15: Let X and Y be two normed linear spaces and $T: D(T) \rightarrow Y$ be a linear operator, where

$D \subset X$ Then the operator T is said to be bounded, if there exists a positive real k such that (Khan et al., 2015)

$$\|Tx\| \le k\|x\|, for\ all\ x \in D(T).$$

The set of all bounded linear operators $B(X; Y)$ (Kreyszig, 1989) is a normed linear space normed by

$$\|T\| = \sup_{x\in X, \|x\|=1} \|Tx\|$$

and $B(X, Y)$ is a Banach space if Y is a Banach space.

Definition 1.16: Let X and Y be two normed linear spaces. An operator $T : X \to Y$ is said to be a compact linear operator (or completely continuous linear operator) if (Khan et al., 2015),

1. T is linear,
2. T maps every bounded sequence (x_k) in X on to a sequence $(T(x_k))$ in Y which has a convergent subsequence.

The set of all compact linear operators $C(X,Y)$ is a closed subspace of $B(X,Y)$ and $C(X,Y)$ is Banach space, if Y is a Banach space.

Definition 1.17: Let X is a vector space. A function $p : X \to \mathbb{R}$ is said to be a paranorm if p satisfies the following conditions:

1. $p(x) \ge 0$, for all $x \in X$,
2. $p(-x) = p(x)$, for all $x \in X$,
3. $p(x + y) \le p(x) + p(y)$, for all $x, y \in X$,
4. if (λ_n) is a sequence of scalars with $(\lambda_n) \to (\lambda)$ as $(n \to \infty)$ and (x_n) is a sequence

of vectors such that $p(x_n - x) \to 0$ as $(n \to \infty)$ then $p(\lambda_n x_n - \lambda x) \to 0$ as $n \to \infty$.

In this article we introduce the following sequence spaces:

$S^I_{(\mu,\nu)}(T)(p) = \{(x_k) \in \ell_\infty : \{k \in \mathbb{N} : \mu[T(x_k - L, t)]^{p_k} \le 1 - \epsilon \text{ Or } \nu[T(x_k) - L, t]^{p_k} \ge \epsilon\} \in I\}$,

$S^I_{0(\mu,\nu)}(T)(p) = \{(x_k) \in \ell_\infty : \{k \in \mathbb{N} : \mu[T(x_k), t]^{p_k} \le 1 - \epsilon \text{ Or } \nu[T(x_k), t]^{p_k} \ge \epsilon\} \in I\}$.

We also define an open ball with center x and radius r with respect to t as follows:

$$B_X(r,t)(T)(p) = \left\{(y_k) \in \ell_\infty : \left\{\begin{matrix} k \in \mathbb{N} : \mu[T(x_k) - T(y_k), t]^{p_k} \le 1 - \epsilon \\ \text{Or } \nu[T(x_k) - T(y_k), t]^{p_k} \ge \epsilon \end{matrix}\right\} \in I\right\}.$$

2. Main results

Theorem 2.1: $S^I_{(\mu,\nu)}(T)(p)$ and $S^I_{0(\mu,\nu)}(T)(p)$ are vector spaces.

Proof: We shall prove the result for $S^I_{(\mu,\nu)}(T)(p)$. The proof for the other space will follow similarly.

Let $x = (x_k), y = (y_k) \in S^I_{(\mu,\nu)}(T)(p)$ and α, β be scalars. Then for given $\epsilon > 0$, we have

$$A_1 = \left\{k \in \mathbb{N} : \mu\left[T(x_k) - L_1, \tfrac{t}{2|\alpha|}\right]^{p_k} \le 1 - \epsilon \ or\ \nu\left[T(x_k) - L_1, \tfrac{t}{2|\alpha|}\right]^{p_k} \ge \epsilon\right\} \in I,$$

$$A_2 = \left\{k \in \mathbb{N} : \mu\left[T(y_k) - L_2, \tfrac{t}{2|\beta|}\right]^{p_k} \le 1 - \epsilon \ or\ \nu\left[T(y_k) - L_2, \tfrac{t}{2|\beta|}\right]^{p_k} \ge \epsilon\right\} \in I.$$

$$A_1^c = \left\{k \in \mathbb{N} : \mu\left[T(x_k) - L_1, \tfrac{t}{2|\alpha|}\right]^{p_k} > 1 - \epsilon \ or\ \nu\left[T(x_k) - L_1, \tfrac{t}{2|\alpha|}\right]^{p_k} < \epsilon\right\} \in \mathcal{F}(I)$$

$$A_2^c = \left\{k \in \mathbb{N} : \mu\left[T(y_k) - L_2, \tfrac{t}{2|\beta|}\right]^{p_k} > 1 - \epsilon \ or\ \nu\left[T(y_k) - L_2, \tfrac{t}{2|\beta|}\right]^{p_k} < \epsilon\right\} \in \mathcal{F}(I).$$

Define the set $A3 = A1 \cup A2$, so that $A3 \in I$. It follows that A_3^c is a non-empty set in $\mathcal{F}(I)$. We shall show that for each $(x_k), (y_k) \in S^I_{(\mu,\nu)}(T)(p)$,

$$A_3^c \subset \left\{\begin{matrix} \mu[\alpha T(x_k) + \beta T(y_k) - (\alpha L_1 + \beta L_2), t]^{p_k} > 1 - \epsilon \\ \text{or} \\ \nu[\alpha T(x_k) + \beta T(y_k) - (\alpha L_1 + \beta L_2), t]^{p_k} < \epsilon \end{matrix}\right\}.$$

Let $m \in A_3^c$, then we have,

$$\mu\left[T(x_m) - L_1, \tfrac{t}{2|\alpha|}\right]^{p_k} > 1 - \epsilon \text{ or } \nu\left[T(x_m) - L_1, \tfrac{t}{2|\alpha|}\right]^{p_k} < \epsilon$$

and

$$\mu\left[T(y_m) - L_2, \tfrac{t}{2|\beta|}\right]^{p_k} > 1 - \epsilon \ or\ \nu\left[T(y_m) - L_2, \tfrac{t}{2|\beta|}\right]^{p_k} < \epsilon.$$

We have

$$\mu[(\alpha T(x_m) - \beta T(y_m)) - (\alpha L_1 + \beta L_2)]^{p_k} \ge \mu\left[\alpha T(x_m) - \alpha L_1, \tfrac{t}{2}\right]^{p_k} * \mu\left[\beta T(x_m) - \beta L_2, \tfrac{t}{2}\right]^{p_k}$$
$$= \mu\left[T(x_m) - L_1, \tfrac{t}{2|\alpha|}\right]^{p_k} * \mu\left[T(x_m) - L_2, \tfrac{t}{2|\beta|}\right]^{p_k}$$
$$> (1 - \epsilon) * (1 - \epsilon) = 1 - \epsilon.$$

and

$$\nu[\alpha T(x_m) + \beta T(y_m) - (\alpha L_1 + \beta L_2), t]^{p_k} \le \nu\left[\alpha T(x_m) - \alpha L_1, \tfrac{t}{2}\right]^{p_k} \circ \nu\left[\beta T(x_m) - \beta L_2, \tfrac{t}{2}\right]^{p_k}$$
$$= \nu\left[T(x_m) - \alpha L_1, \tfrac{t}{2|\alpha|}\right]^{p_k} \circ \nu\left[T(x_m) - L_2, \tfrac{t}{2|\beta|}\right]^{p_k}$$
$$< \epsilon \circ \epsilon = \epsilon.$$

Therefore

$$A_3^c \subset \left\{\begin{matrix} k \in \mathbb{N} : \mu[\alpha T(x_k) + \beta T(y_k) - (\alpha L_1 + \beta L_2), t]^{p_k} > 1 - \epsilon \\ or\ \nu[\alpha T(x_k) + \beta T(y_k) - (\alpha L_1 + \beta L_2), t]^{p_k} < \epsilon \end{matrix}\right\} = B(say).$$

So we have $B^c \subset A_3 \in I$ which proves $S^I_{(\mu,\nu)}(T)(p)$ is a linear space.

Theorem 2.2: Every open ball $B_x(r,t)(T)(p)$ is an open set in $S^I_{(\mu,\nu)}(T)(p)$.

Proof: Let $B_x(r,t)(T)(\mathcal{P})$ be an open ball with center x and radius r with respect to t. That is

$$B_x(r,t)(T)(p) = \left\{ \begin{array}{l} y=(y_k)\in \ell_\infty : \{k\in \mathbb{N}: \mu[T(x_k)-T(y_k),t]^{p_k} \le 1-r \\ \quad or\ \nu[T(x_k)-T(y_k),t]^{p_k} \ge r\} \in I \end{array} \right\}$$

Let $y \in B^c_x(r,t)(T)(p)$. Then

$\mu[T(x_k)-T(y),t]^{p_k} > 1-r$ and $\nu[T(x_k)-T(y_k),t]^{p_k} < r$.

Since $\mu[T(x_k)-T(y_k),t]^{p_k} > 1-r$ there exists $t_0 \in (0,1)$ such that $\mu[T(x_k)-T(y_k),t_0]^{p_k} > 1-r$ and $\nu[T(x_k)-T(y_k),t_0]^{p_k} < r$. Putting $r_o = \mu[T(x_k)-T(y_k),t_0]^{p_k}$, we have $r_0 > 1-r$ there exists $s \in (0,1)$ such that $s<r$ and hence $r_0 > 1-s > 1-r$. For $r_0 > 1-s$, we have $r_1, r_2 \in (0,1)$ with $r_1, r_2 > r_0$ and thus $r_0 * r_1 > 1-s$ and $(1-r_0) \diamond (1-r_0) \le s$. Let $r_3 = \max\{r_1, r_2\}$ and consider the ball $B^c_y(1-r_3, t-t_0)(T)(p)$. We proof that

$$B^c_y(1-r_3,t-t_0)(T)(p) \subset B^c_x(r,t)(T)(p).$$

Let $z=(z_k) \in B^c_y(1-r_3,t-t_0)(T)(p)$. Then
$\mu[T(y_k)-T(z_k),t-t_0]^{p_k} > r_3$ and $\nu[T(y_k)-T(z_k),t-t_0]^{p_k} < 1-r_3$.

Thus

$$\mu[T(x_k)-T(z_k),t]^{p_k} > \mu[T(x_k)-T(y_k),t_0]^{p_k} * \mu[T(y_k)-T(z_k),t-t_0]^{p_k}$$
$$\ge (r_0 * r_3) \ge (r_0 * r_1) \ge (1-s) > (1-r)$$

and

$$\nu[T(x_k)-T(z_k),t]^{p_k} \le \nu[T(x_k)-T(y_k),t_0]^{p_k} \diamond \nu[T(y_k)-T(z_k),t-t_0]^{p_k}$$
$$\le (1-r_0)\diamond(1-r_3) \le (1-r_0)\diamond(1-r_2) \le s < r.$$

Thus $z \in B^c_x(r,t)(T)(p)$ and hence

$$B^c_y(1-r_3,t-t_0)(T)(p) \subset B^c_x(r,t)(T)(p).$$

Remark 2.1: $S^I_{(\mu,\nu)}(T)(p)$ is an IFNS.

Define $\tau^I_{(\mu,\nu)}(T)(p) = \{A \subset S^I_{(\mu,\nu)}(T)(p)$: for each $x \in A$ there exists $t>0$ and $r \in (0,1)$. Such that $B_x(r,t)(T)(p) \subset A\}$. Then $\tau^I_{(\mu,\nu)}(T)(p)$ is a topology on $S^I_{(\mu,\nu)}(T)(p)$.

Theorem 2.3: The topology $\tau^I_{(\mu,\nu)}(T)(p)$ on $S^I_{0(\mu,\nu)}(T)(p)$ is first countable.

Proof: $\left\{B_x\left(\frac{1}{n},\frac{1}{n}\right)(T)(p): n=1,2,3,\dots\dots\right\}$ is a local base at x. Hence the topology $\tau^I_{(\mu,\nu)}(T)(p)$ on$S^I_{0(\mu,\nu)}(T)(p)$ is first countable.

Theorem 2.4: $S^I_{(\mu,\nu)}(T)(p)$ and $S^I_{0(\mu,\nu)}(T)(p)$ are Hausdorff spaces.

Proof: We prove the result for $S^I_{(\mu,\nu)}(T)(p)$. The proof for $S^I_{0(\mu,\nu)}(T)(p)$ will follow on similar lines. Let $x,y \in S^I_{(\mu,\nu)}(T)(p)$ such that $x \ne y$. Then $0 < \mu[T(x)-T(y)]^{p_k} < 1$ and $0 < \nu[T(x)-T(y)]^{p_k} < 1$. Put $r_1 = \mu[T(x)-T(y)]^{p_k}$, $r_2 = \nu[T(x)-T(y),t]^{p_k}$ and $r = \max\{r_1, 1-r_2\}$. For each $r_0 \in (r,1)$ There exists r_3 and r_4 with $r_3, r_4 > r$ and hence $r_3 * r_4 \ge r_0$ and $(1-r_3)\diamond(1-r_4) \le (1-r_0)$. Let $r_5 = \max\{r_3, 1-r_4\}$, then we can show the open balls $B^c_x\left(1-r_5,\frac{t}{2}\right)$ and $B^c_y\left(1-r_5,\frac{t}{2}\right)$ are disjoint. Suppose on contrary there exists

$$z \in B^c_x\left(1-r_5,\frac{t}{2}\right) \cap B^c_y\left(1-r_5,\frac{t}{2}\right),$$

then

$$r_1 = \mu[T(x)-T(y),t]^{p_k} \ge \mu\left[T(x)-T(z),\frac{t}{2}\right]^{p_k} * \mu\left[T(z)-T(y),\frac{t}{2}\right]^{p_k}$$
$$> r_5 * r_5 \ \ge r_3 * r_3 \ge r_0 > r_1$$

and

$$r_2 = \nu[T(x)-T(y),t]^{p_k} \le \nu\left[T(x)-T(z),\frac{t}{2}\right]^{p_k} \diamond \nu\left[T(z)-T(y),\frac{t}{2}\right]^{p_k}$$
$$< (1-r_5)\diamond(1-r_5) \le (1-r_3)\diamond(1-r_3)$$
$$\le (1-r_0) < r_2$$

which is contradiction. Hence $S^I_{(\mu,\nu)}(T)(p)$ is Hausdorff spaces.

Theorem 2.5: $S^I_{(\mu,\nu)}(T)(p)$ is IFNS and $\tau^I_{(\mu,\nu)}(T)(p)$ is topology on $S^I_{(\mu,\nu)}(T)(p)$. Then a sequence $x=(x_k) \in S^I_{(\mu,\nu)}(T)(p)$, $x_k \to x$ if and only if $\mu[T(x_k)-T(x),t]^{p_k} \to 1$ and

$\nu[T(x_k)-T(x),t]^{p_k} \to 0$ as $k \to \infty$.

Proof: Fix $t_0 > 0$. suppose $x_k \to x$ as $(k \to \infty)$. Then for $r \in (0,1)$, there exists $n_0 \in \mathbb{N}$ such that $(x_k) \in B_x(r,t)(T)(p)$ for all $k \ge n_0$.
$B_x(r,t)(T)(p) = \{k \in \mathbb{N}: \mu[T(x_k)-T(x),t]^{p_k} \le 1-r$ or $\nu[T(x_k)-T(x),t]^{p_k} \ge r\} \in I$,

and so $B^c_x(r,t)(T)(p) \in F(I)$. Then $1-\mu[T(x_k)-T(x),t]^{p_k} < r$ and $\nu[T(x_k)-T(x),t]^{p_k} < r$. Hence $\mu[T(x_k)-T(x),t]^{p_k} \to 1$ and $\nu[T(x_k)-T(x),t]^{p_k} \to 0$ as $k \to \infty$.

Conversely, if for each $t>0$ $\mu[T(x_k)-T(x),t]^{p_k} \to 1$ and $\nu[T(x_k)-T(x),t]^{p_k} \to 0$ as $k \to \infty$, then $r \in (1,0)$, there exists $n_0 \in \mathbb{N}$ such that $1-\mu[T(x_k)-T(x),t]^{p_k} < r$ and $\nu[T(x_k)-T(x),t]^{p_k} < r$ for all $k \ge n_0$. It follows that $\mu[T(x_k)-T(x),t]^{p_k} > 1-r$ and $\nu[T(x_k)-T(x),t]^{p_k} < r$ for all $k \ge n_0$. Thus $(x_k) \in B^c_x(r,t)(T)(p)$ for all $k \ge n_0$ and hence $x_k \to x$.

Theorem 2.6: A Sequence $x = (x_k) \in S^I_{(\mu,\nu)}(T)(p)$ is I-convergent if and only if for every $\epsilon > 0$ and $t > 0$ there exists a number $N = N(x, \epsilon, t)$ such that

$$\left\{k \in \mathbb{N}: \mu \left[T\ (x_k) - L, \frac{t}{2}\right]^{p_k} > 1 - \epsilon \text{ or } \nu \left[T\ (x_k) - L, \frac{t}{2}\right]^{p_k} < \epsilon\right\} \in \mathcal{F}(I)$$

Proof: Suppose that $I_{(\mu,\nu)} - \lim x = L$ and let $\epsilon > 0$ and $t > 0$. For a given $\epsilon > 0$, choose $s > 0$ such that $(1-\epsilon) * (1-\epsilon) > 1 - s$ and $\epsilon \diamond \epsilon < s$. Then for each $x \in S^I_{(\mu,\nu)}(T)(p)$

$$A = \left\{k \in \mathbb{N}: \mu \left[T\ (x_k) - L, \frac{t}{2}\right]^{p_k} \le 1 - \epsilon \text{ or } \nu \left[T\ (x_k) - L, \frac{t}{2}\right]^{p_k} \ge \epsilon\right\} \in I$$

and thus

$$A^c = \left\{k \in \mathbb{N}: \mu \left[T\ (x_k) - L, \frac{t}{2}\right]^{p_k} > 1 - \epsilon \text{ or } \nu \left[T\ (x_k) - L, \frac{t}{2}\right]^{p_k} < \epsilon\right\} \in \mathcal{F}(I).$$

Conversely let us choose $N \in A$. Then

$$\mu \left[T\ (x_N) - L, \frac{t}{2}\right]^{p_k} > 1 - \epsilon \text{ or } \nu \left[T\ (x_N) - L, \frac{t}{2}\right]^{p_k} < \epsilon.$$

Now we want to show that there exists number $N = N(x, \epsilon, t)$ such that

$$\{k \in \mathbb{N}: \mu\ [T\ (x_k) - T\ (x_N), t]^{p_k} \le 1 - s \text{ or } \nu[T\ (x_k) - T\ (x_N), t]^{p_k} \ge s\} \in I.$$

For this, define for each $x \in S^I_{(\mu,\nu)}(T)(p)$

$$B = \{k \in \mathbb{N}: \mu\ [T\ (x_k) - T\ (x_N), t]^{p_k} \le 1 - s \text{ or } \nu[T\ (x_k) - T\ (x_N), t]^{p_k} \ge s\}.$$

Now we have to show that $B \subset A$. Suppose $B \subsetneq A$. Then there exists $n \in B$ and $n \notin A$. Therefore we have,

$$\mu\ [T\ (x_n) - T\ (x_N), t]^{p_k} \le 1 - s \text{ and } \mu \left[T\ (x_n) - L, \frac{t}{2}\right]^{p_k} > 1 - \epsilon.$$

In particular $\mu \left[T\ (x_N) - L, \frac{t}{2}\right]^{p_k} > 1 - \epsilon$. Thus,

$$1 - s \ \ge \mu\ [T\ (x_n) - T\ (x_N), t]^{p_k} \ge \ \mu \left[T\ (x_n) - L, \frac{t}{2}\right]^{p_k} * \mu \left[T\ (x_n) - L, \frac{t}{2}\right]^{p_k} > (1-\epsilon) * (1-\epsilon) > 1 - s.$$

Which is not possible. Also we have,

$$\nu\ [T\ (x_n) - T\ (x_N), t]^{p_k} \ge s \text{ and } \nu \left[T\ (x_n) - L, \frac{t}{2}\right]^{p_k} < \epsilon.$$

In particular $\nu \left[T\ (x_N) - L, \frac{t}{2}\right]^{p_k} < \epsilon$. Thus,

$$s \ \le \mu\ [T\ (x_n) - T\ (x_N), t]^{p_k} \le \ \nu \left[T\ (x_n) - L, \frac{t}{2}\right]^{p_k} \diamond \nu \left[T\ (x_N) - L, \frac{t}{2}\right]^{p_k} < \epsilon \diamond \epsilon < s,$$

which is not possible. Hence $B \subset A$. $A \in I$ implies $B \in I$.

3. Conclusion

Fuzzy set theory is a powerful hand set for modelling uncertainty and vagueness in various problems arising in field of science and engineering. It has a wide range of applications in various fields. The concept of intuitionistic fuzzy normed space and of intuitionistic fuzzy 2-normed space is the latest developments in fuzzy topology. In the present paper we studied a more general type of paranorm intuitionistic fuzzy I-convergent sequence spaces defined by compact operator and study the fuzzy topology on the said spaces. These results provide new tools to deal with the I-convergence in intuitionistic fuzzy problems of sequences occurring in many branches of science and engineering.

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